Signal-to-Noise as a Function of Chopping and Nodding

## James De Buizer

SOFIA-USRA

August 2, 2010

On-Chip vs. Off-Chip Chop-Nod

- **Chop-nodding completely off-chip or on-chip yields the same signal-to-noise** in the same amount of observing time.

- The source must be located in the same place on the array in both nod beams in the off-chip case for this to hold (otherwise you do lose a factor of root 2 chopping off-chip).

- The following examples show visually and mathematically how these two chop-nod scenarios yield the same signal-to-noise in the background noise-limited case.
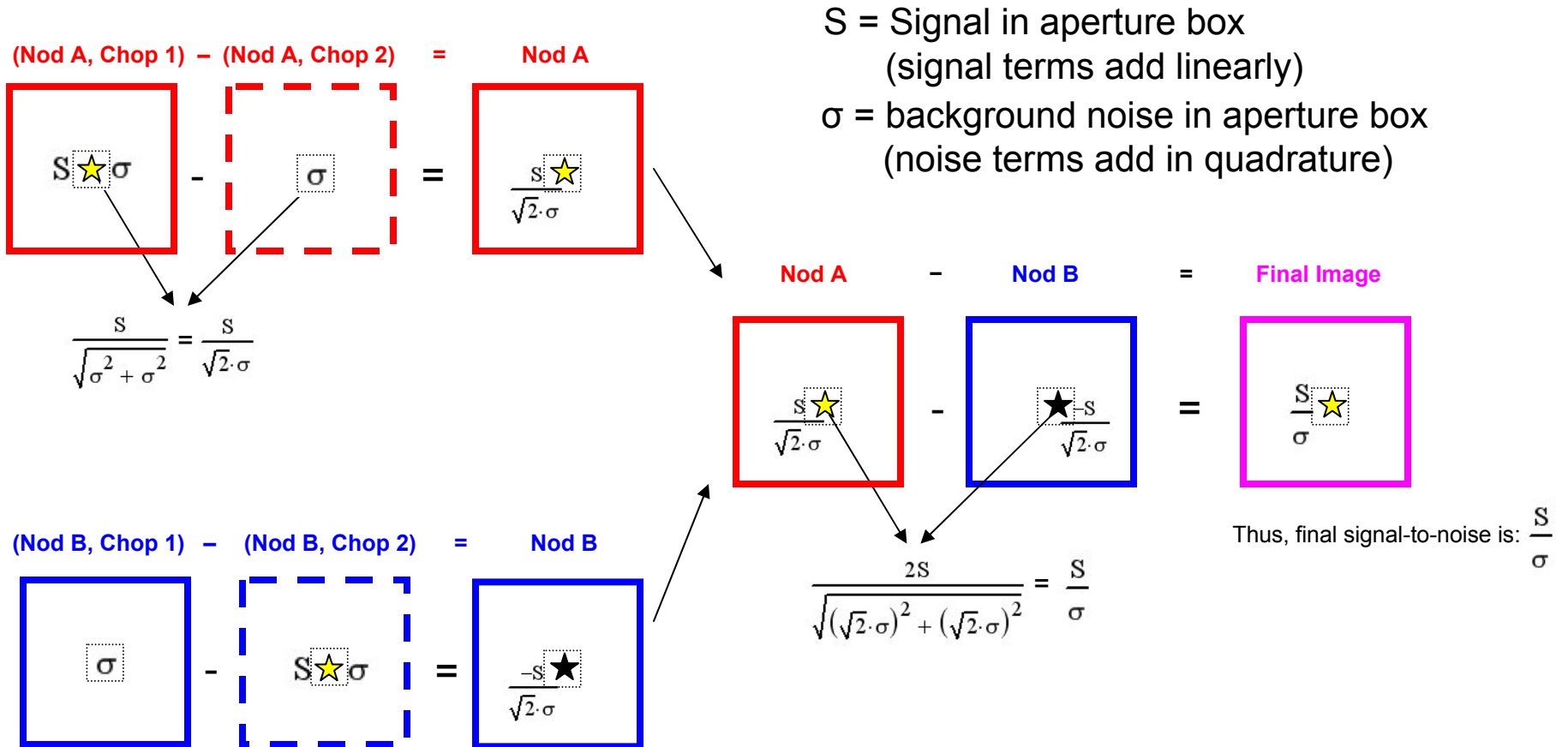
Case #1: Chop-Nod Off-Chip

How chop-nod looks on sky:
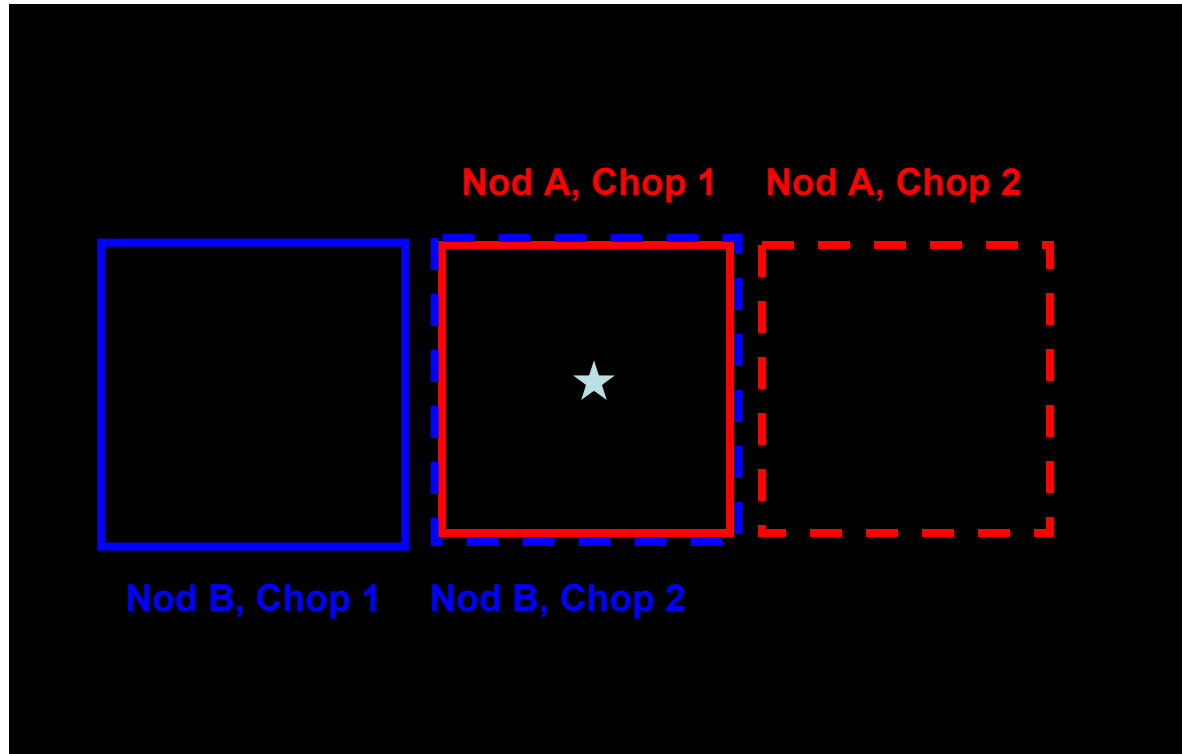
Case #1: Chop-Nod Off-Chip

Following the signal and noise in a small aperture on the frame (dashed boxes):

S = Signal in aperture box
(signal terms add linearly)

σ = background noise in aperture box
(noise terms add in quadrature)

(Nod A, Chop 1) − (Nod A, Chop 2) = Nod A

$$S \bigstar \sigma \quad - \quad \sigma \quad = \quad \frac{S \bigstar}{\sqrt{2} \cdot \sigma}$$

$$\frac{S}{\sqrt{\sigma^2 + \sigma^2}} = \frac{S}{\sqrt{2} \cdot \sigma}$$

Nod A − Nod B = Final Image

$$\frac{S \bigstar}{\sqrt{2} \cdot \sigma} \quad - \quad \frac{\bigstar -S}{\sqrt{2} \cdot \sigma} \quad = \quad \frac{S \bigstar}{\sigma}$$

Thus, final signal-to-noise is: $\dfrac{S}{\sigma}$

(Nod B, Chop 1) − (Nod B, Chop 2) = Nod B

$$\sigma \quad - \quad S \bigstar \sigma \quad = \quad \frac{-S \bigstar}{\sqrt{2} \cdot \sigma}$$

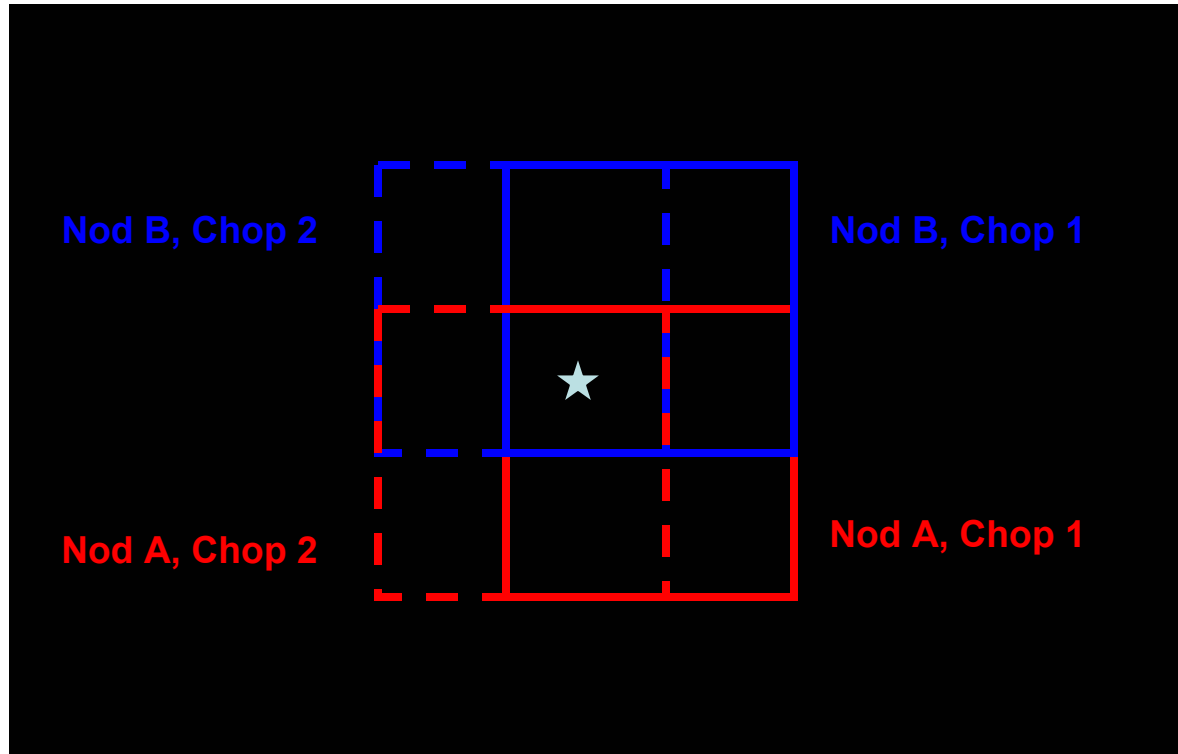$$\frac{2S}{\sqrt{\left(\sqrt{2} \cdot \sigma\right)^2 + \left(\sqrt{2} \cdot \sigma\right)^2}} = \frac{S}{\sigma}$$

Note: Because the noise, σ, on the frames is background noise-limited, the σ in the apertures containing the source signal is equal to the σ in the aperture without the source present (i.e., there is no significant additional source photon noise in the apertures containing the source signal).

Case #2: Chop-Nod On-Chip
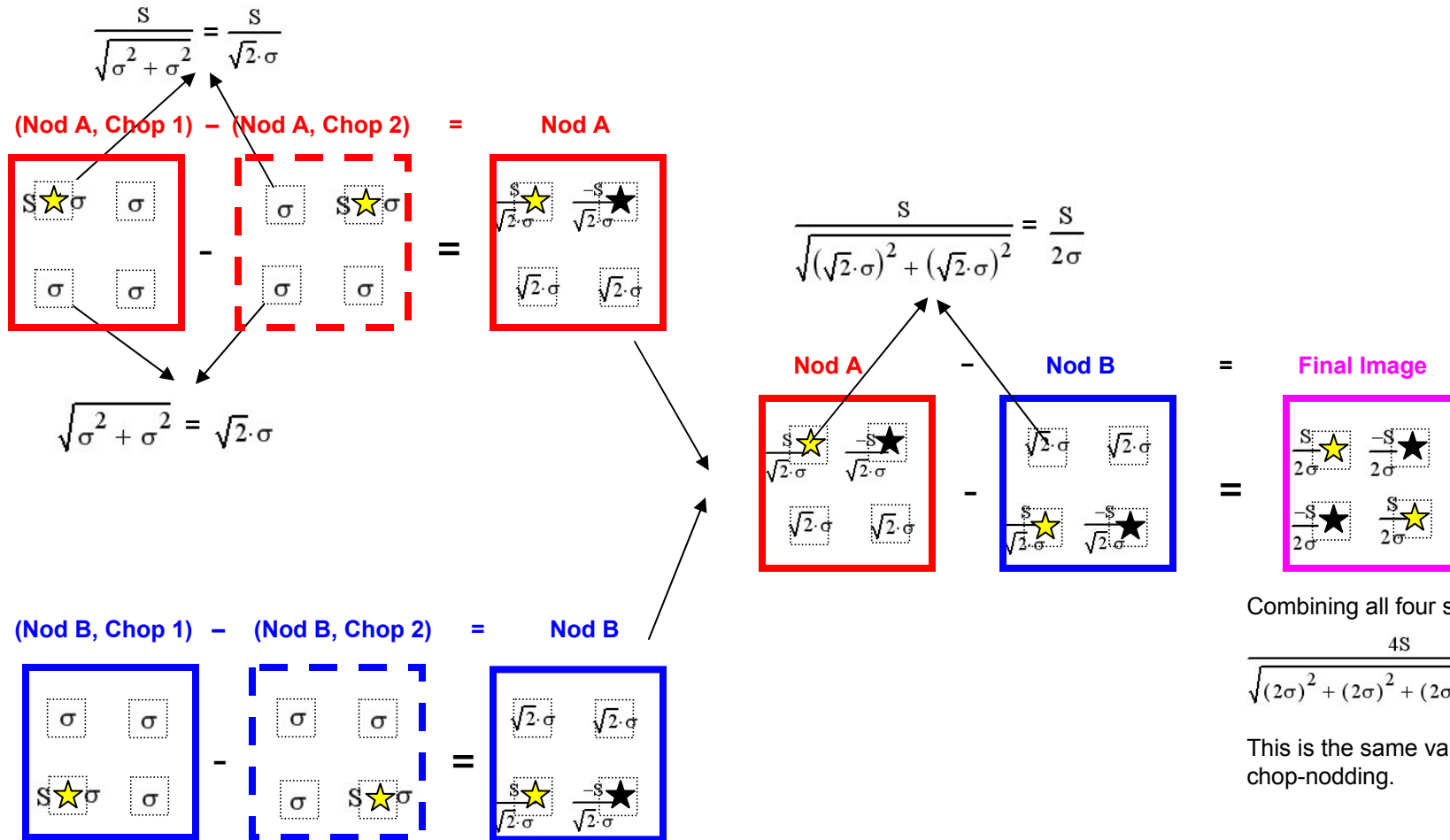## Nodding Perpendicular to Chop Case

How chop-nod looks on sky:

Case #2: Chop-Nod On-Chip
Nodding Perpendicular to Chop Case

Again, following the signal and noise in a small aperture on the frame (dashed boxes):

$$\frac{S}{\sqrt{\sigma^2 + \sigma^2}} = \frac{S}{\sqrt{2}\cdot\sigma}$$

(Nod A, Chop 1) − (Nod A, Chop 2) = Nod A

$$\sqrt{\sigma^2 + \sigma^2} = \sqrt{2}\cdot\sigma$$

$$\frac{S}{\sqrt{\left(\sqrt{2}\cdot\sigma\right)^2 + \left(\sqrt{2}\cdot\sigma\right)^2}} = \frac{S}{2\sigma}$$

Nod A − Nod B = Final Image

Combining all four sources yields:

$$\frac{4S}{\sqrt{(2\sigma)^2 + (2\sigma)^2 + (2\sigma)^2 + (2\sigma)^2}} = \frac{S}{\sigma}$$

This is the same value as for off-chip chop-nodding.

(Nod B, Chop 1) − (Nod B, Chop 2) = Nod B

Note: Again, because the noise, σ, on the frames is background noise-limited, the σ in the apertures containing the source signal is equal to the σ in the aperture without the source present (i.e., there is no significant additional source photon noise in the apertures containing the source signal).

Case #3: Chop-Nod On-Chip
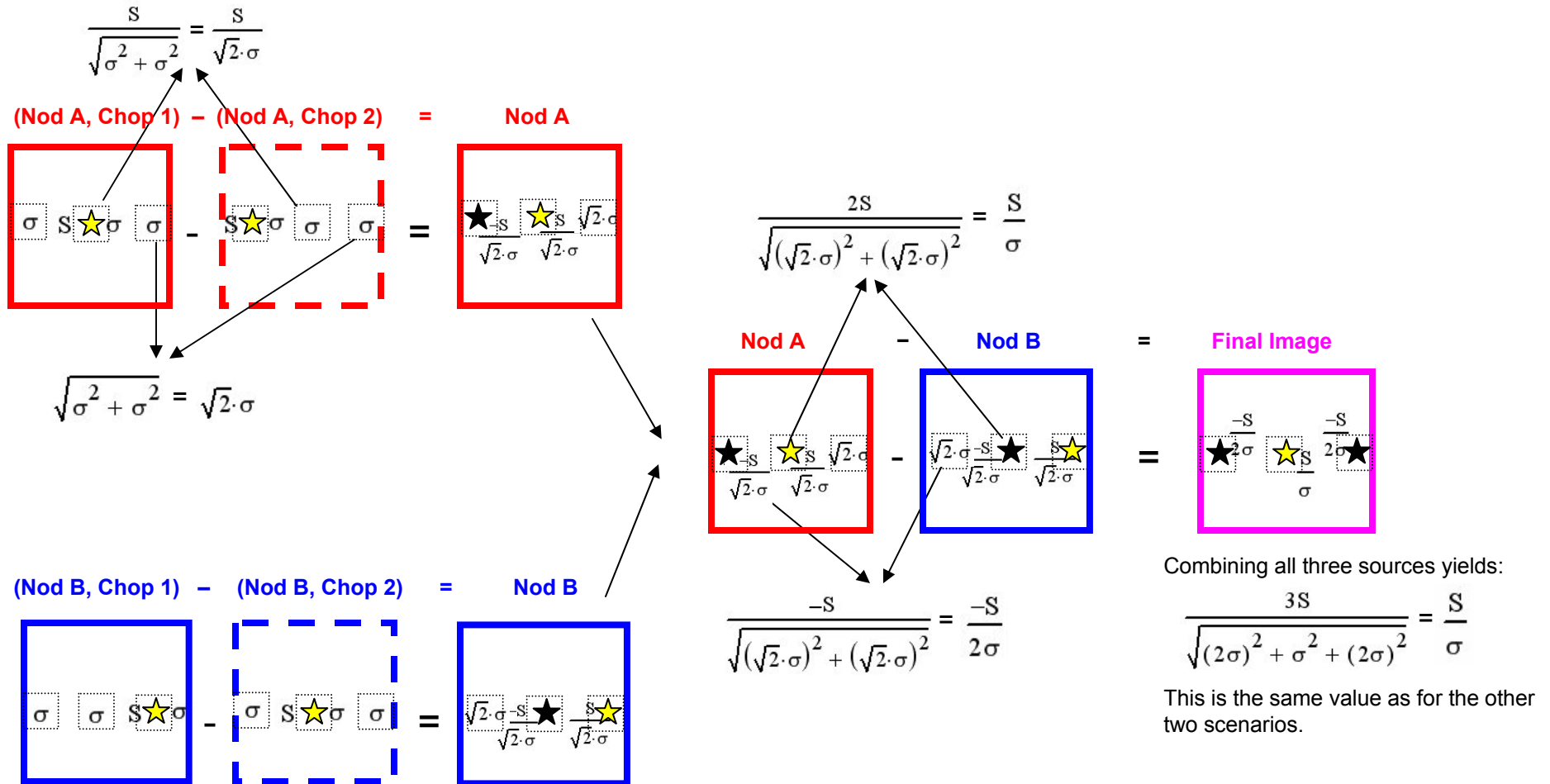## Nodding Matching Chop Case

How chop-nod looks on sky:
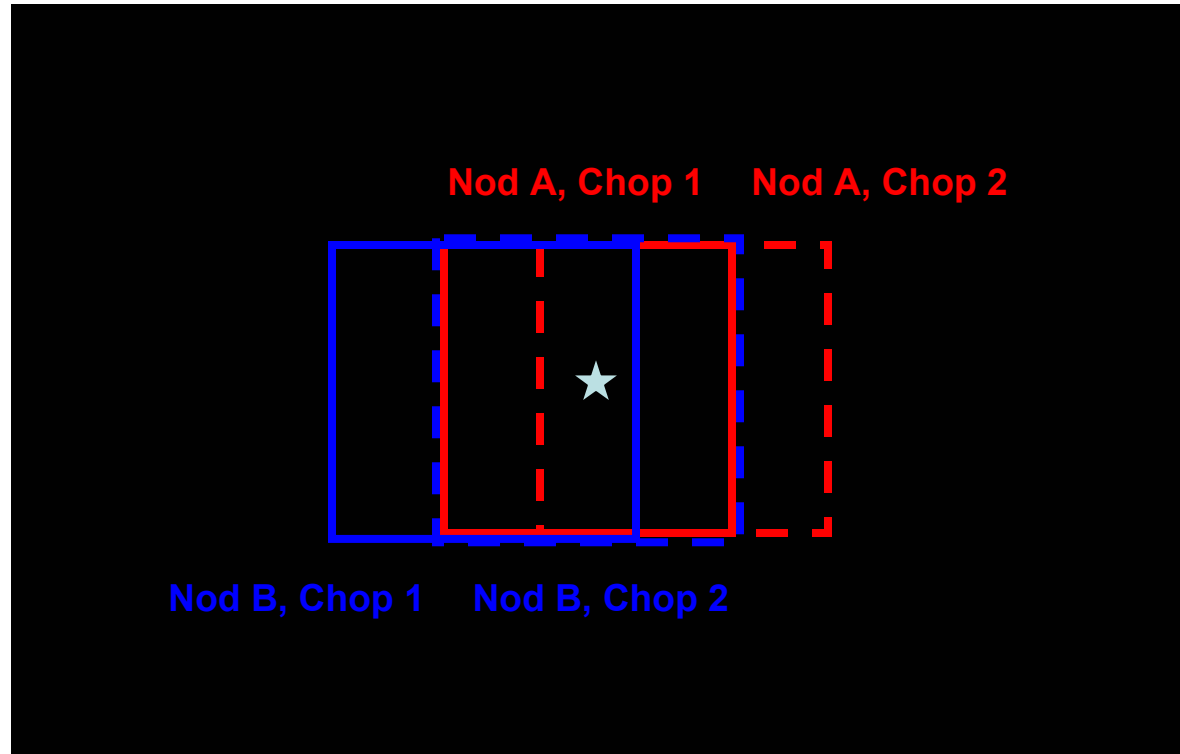
Note: This set-up is very similar to Case #1, except chops and nods are much shorter so that chop reference beams will still be on the array, creating "negative" sources.

Case #3: Chop-Nod On-Chip
Nodding Matching Chop Case

Again, following the signal and noise in a small aperture on the frame (dashed boxes):

$$\frac{S}{\sqrt{\sigma^2 + \sigma^2}} = \frac{S}{\sqrt{2} \cdot \sigma}$$

(Nod A, Chop 1) − **(Nod A, Chop 2)** = **Nod A**

$$\sqrt{\sigma^2 + \sigma^2} = \sqrt{2} \cdot \sigma$$

$$\frac{2S}{\sqrt{(\sqrt{2} \cdot \sigma)^2 + (\sqrt{2} \cdot \sigma)^2}} = \frac{S}{\sigma}$$

Nod A − **Nod B** = **Final Image**

$$\frac{-S}{\sqrt{(\sqrt{2} \cdot \sigma)^2 + (\sqrt{2} \cdot \sigma)^2}} = \frac{-S}{2\sigma}$$

(Nod B, Chop 1) − **(Nod B, Chop 2)** = **Nod B**

Combining all three sources yields:

$$\frac{3S}{\sqrt{(2\sigma)^2 + \sigma^2 + (2\sigma)^2}} = \frac{S}{\sigma}$$

This is the same value as for the other two scenarios.

Intuitively, it may seem that there should be an S/N advantage over Case #1 by being able to add in the flux from the negative sources on-field. However, these calculations show that any additional signal from these negative sources are negated by the additional higher noise associated with them. In other words, combining the signal and noise from all three sources has no advantage over ignoring the negative sources and just using the central source alone.